On S-duality and 6d CFTs

Seok Kim

(Seoul National University)

Autumn symposium on string theory, KIAS

Sep 12, 2017

Talk based on:

SK, June Nahmgoong, “Asymptotic M5-brane entropy from S-duality” 1702.04058.

+ more works in (slow) progress

Some related works:

Billo, Frau, Fucito, Lerda, Morales,
“S-duality and the prepotential in N=2* theories (I): the ADE algebras” 1507.07709.

Haghighat, Iqbal, Kozcaz, Lockhart, Vafa, “M-strings” 1305.6322.

Di Pietro, Komargodski, “Cardy formulae for SUSY theories in d=4 and d=6” 1407.6061.

Galakhov, Mironov, Morozov,
“S-duality & modular transformation as a non-perturbative deformation of the ordinary pq-duality” 1311.7069.

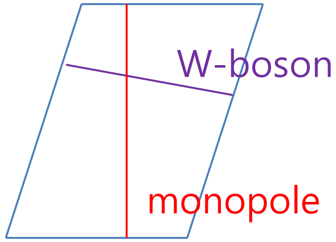
S-duality

- Relation between two interacting QFTs w/ coupling constants inversed.
- 3+1d gauge theories: electric-magnetic duality

- Simple example: maximal super-Yang-Mills [Montonen, Olive] [Osborn]
- Evidences: dyon spectrum [Sen], various partition functions [Vafa, Witten] … , etc.

- Accounted for by embedding into bigger systems
- D3's on type IIB: consequence of S-duality of IIB string theory
- A smaller embedding, in QFT: 6d N=(2,0) QFT (e.g. on M5's) on T^2

complex structure $\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{YM}^2} \sim$ 4d coupling constant



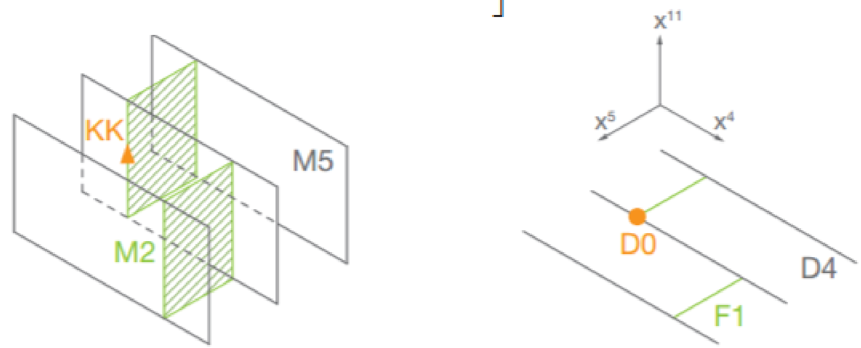
- Tied to subtle properties of 6d QFTs

- 6d (2,0) theory has been a guidance to S-duality. **Can it be true the other way?**

The observable

- N M5's on S^1 : an index for BPS states in the tensor branch. $Z[R^4 \times T^2]$
- bound states of wrapped self-dual strings & momenta

$$Z(\tau, m, \epsilon_{1,2}, v) = \mathrm{Tr}\left[(-1)^F e^{2\pi i\tau\frac{H+P}{2}} e^{-2\pi i\bar{\tau}\frac{H-P}{2}} e^{\epsilon_1(J_1+J_R)+\epsilon_2(J_2+J_R)} e^{2mJ_L} e^{-v_i q_i}\right]$$



- IIA picture: k D0's bound to N D4's & F1's

$$Z(\tau, m, \epsilon_{1,2}, v) = Z_{\mathrm{pert}}(m, \epsilon_{1,2}, v) \sum_{k=0}^{\infty} q^k Z_k(m, \epsilon_{1,2}, v) \qquad Z_k = \sum_{Y_i; \sum_{i=1}^N |Y_i| = k} \prod_{i,j=1}^N \prod_{s \in Y_i} \frac{\sinh\frac{E_{ij}(s)+m-\epsilon_+}{2} \sinh\frac{E_{ij}(s)-m-\epsilon_+}{2}}{\sinh\frac{E_{ij}(s)}{2} \sinh\frac{E_{ij}(s)-2\epsilon_+}{2}}$$

[Nekrasov] (2002) [Bruzzo, Fucito, Morales, Tanzini] (2002)
[Nekrasov, Okounkov] (2003) [H.-C. Kim, SK, E. Koh, K. Lee, S. Lee] (2011)

- Chemical potentials:
- τ: for left-moving momentum. (more on it on next slide)
- v_i: for winding numbers. ~ tensor/Coulomb VEV
- m: for $SU(2)_L \subset SO(5)_R$. ~ mass deformation parameter for MSYM
- $\epsilon_{1,2}$: $SO(4)$ spatial rotations (locked w/ $SU(2)_R \subset SO(5)$). "Ω deformation" (partly IR regulator)

Using S-duality in 6d?

- A partition function on R^4 x T^2 : (spatial S^1) x (temporal S^1)
  - $\tau \sim i\frac{R_t}{R_s} \sim \frac{i}{T\,R_s}$: inverse-temperature (in KK unit). coupling const. in 4d limit
  - finite T^2: Is there S-duality $\tau \to -1/\tau$ (or $R_s \leftrightarrow R_t$) in the partition function?

- Connects "low & high T": useful tool to study "high T" or "decompactifying" regime
  - But if $\log Z \sim N^3$ & $\log Z \sim N^2$ at high/low T, hard to expect exact S-duality.
  - So, natural to expect certain 'anomaly' of S-duality.

- Goal: Explore the 6d extension of 4d S-duality, and use it to study 6d physics.

- Will mostly focus on "prepotential" : $Z(\tau, v, m, \epsilon_{1,2}) \sim \exp[-\frac{f(\tau,v,m)}{\epsilon_1\epsilon_2}]$ at $\epsilon_{1,2} \to 0$.
  - Can be viewed either as Seiberg-Witten action, or free energy at "large volume"
  - Technically easier to study than the full partition function

4d limit: S-duality of $\mathcal{N} = 2^*$ prepotential

- S-duality in SW theory: "magnetic prepotential = same function as electric one"

$$F_D^{\rm 4d}(\tau_D, a_D, M) = \mathcal{L}[F^{\rm 4d}](\tau, a, M) = F^{\rm 4d}(\tau, a, M) - a\frac{\partial F^{\rm 4d}}{\partial a}(\tau, a, M) \qquad a_D = \frac{1}{2\pi i}\frac{\partial F}{\partial a}$$

- Expect S-duality after a suitable a decomposition

$$F^{\rm 4d}(\tau, a, M) = F^{\rm 4d}_{\text{S-dual}}(\tau, a, M) + \boxed{F^{\rm 4d}_{\rm anom}(\tau, M)}$$

independent of Coulomb VEV: ambiguous in SW theory

$$F^{\rm 4d}_{\text{S-dual}}(\tau_D, a_D, M) = F^{\rm 4d}_{\text{S-dual}}(\tau, a, M) - a\frac{\partial F^{\rm 4d}_{\text{S-dual}}}{\partial a}(\tau, a, M)$$

- Tested in small M expansion by summing over $q = e^{2\pi i\tau}$

- $F^{4d}_{S\,dual}(\tau, a, M) = \pi i\tau a^2 + \boxed{f} \equiv \pi i\tau a^2 + \sum_{n=1} M^{2n}\boxed{f_n(\tau, a)}$

"quantum" (=1-loop+instanton) part

can write coefficients exactly in q, using "quasi-modular forms"

[Billo, Frau, Fucito, Lerda, Morales] (2015)

6d extension

- For simplicity, first consider $M \ll a$: classical S-duality $\quad a_D = \tau a + \cancel{\frac{1}{2\pi i}\frac{\partial f}{\partial a}}$
- Good in 4d: Scalar eigenvalues classically live on $C \sim R^2$.
- 6d: Scalars live on $R \times S^1$. Unnatural to rotate w/ complex τ.

6d tensor VEV (noncompact)

$$a \sim R_s(\Phi + iB_{12})$$

6d 2-form on T^2 (compact)

- Natural dimensionless variables (~ chemical potentials)

redefine to dimensionless f by absorbing R_t^2 factor

$$v \equiv R_t a\ , \quad v_D \equiv \frac{R_t a_D}{\tau} = v + \frac{1}{2\pi i\tau}\frac{\partial(R_t^2 f)}{\partial v} \to v + \frac{1}{2\pi i\tau}\frac{\partial f}{\partial v} \qquad m = M\,R_t\ ,\ \mathrm{v} = a\,R_t$$

- 6d version of S-duality: using dimensionless variables & f (correctly reduces to 4d ones)

$$\tau^2 f\left(-\frac{1}{\tau},\ v + \frac{1}{2\pi i\tau}\frac{\partial f}{\partial v},\ \frac{m}{\tau}\right) = f(\tau, v, m) + \frac{1}{4\pi i\tau}\left(\frac{\partial f}{\partial v}(\tau, v, m)\right)^2$$

- Checked in same way, expanding in small m: curious 'anomaly' in S-duality

$$F_{\rm anom} = N f_{U(1)}(\tau, m) + \frac{N^3 - N}{288} m^4 E_2(\tau) \rightarrow$$

After restoring R_t, the 2nd term $\propto R_t^2 M^4 (N^3 - N) \to 0$ disappears in 4d limit

- $F_{anom}(\tau, m)$ meaningless in SW theory. But in free energy, neutral states' contribution.

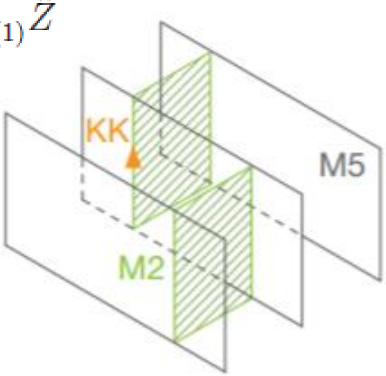
[Nahmgoong, SK] (2017)

Alternative derivation

- Expand Z w/ tensor VEVs, or fugacities e^{-v_i} for string winding #'s

$$Z(\tau, v, m, \epsilon_{1,2}) = e^{-\varepsilon_0} Z_{U(1)}(\tau, m, \epsilon_{1,2})^N \sum_{n_1,\cdots,n_{N-1}=0}^{\infty} e^{-\sum_{i=1}^{N-1} n_i \alpha_i(v)} Z_{(n_i)}(\tau, m, \epsilon_{1,2}) \equiv e^{-\varepsilon_0} Z_{U(1)}^N \hat{Z}$$

simple roots



- $Z_{(n_i)}(\tau, m, \epsilon_{1,2})$: elliptic genus of self-dual strings

  [Haghighat, Iqbal, Kozcaz, Lockhart, Vafa] (2013)

$$Z_{(n_i)} = \sum_{Y_1,\cdots,Y_{N-1};|Y_i|=n_i} \prod_{i=1}^{N} \prod_{s\in Y_i} \frac{\theta_1(\tau|\frac{E_{i,i+1}(s)-m+\epsilon_-}{2\pi i})\theta_1(\tau|\frac{E_{i,i-1}(s)+m+\epsilon_-}{2\pi i})}{\theta_1(\tau|\frac{E_{i,i}(s)+\epsilon_1}{2\pi i})\theta_1(\tau|\frac{E_{i,i}(s)-\epsilon_2}{2\pi i})}$$

- Modular anomaly of elliptic genus $Z_{(n_i)}(\tau, m, \epsilon_{1,2})$: [HIKLV] (2013)

$$\frac{\theta_1(-\frac{1}{\tau}, \frac{z}{\tau})}{\eta(-\frac{1}{\tau})} = e^{\frac{\pi i z^2}{\tau}} \frac{\theta_1(\tau, z)}{\eta(\tau)}$$

$$Z_{(n_i)}\left(-\frac{1}{\tau}, \frac{m}{\tau}, \frac{\epsilon_{1,2}}{\tau}\right) = \exp\left[\frac{1}{4\pi i\tau}\left(\epsilon_1\epsilon_2\Omega^{ij}n_i n_j - \Omega^{ij}(2m^2 - 2\epsilon_+^2)\rho_i n_j\right)\right] Z_{n_i}(\tau, m, \epsilon_{1,2})$$

Ω^{ij}: Cartan matrix for A_{N-1} Lie algebra

$\rho = \frac{1}{2}\sum_{\alpha>0}\alpha$: Weyl vector for A_{N-1}

- This determines the S-duality anomaly of $Z(\tau, v, m, \epsilon_{1,2})$, & $f(\tau, v, m)$.

Modular anomaly equation

- τ dependence via quasi-modular forms: $\theta_1(\tau|z) = 2\pi i z\ \eta(\tau)^3 \exp\left[\sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)(2k)!} E_{2k}(\tau)(2\pi i z)^{2k}\right]$

- 3 generators: $E_2(-1/\tau) = \tau^2\left(E_2 + \boxed{\frac{6}{\pi i \tau}}\right)\ ,\quad E_4(-1/\tau) = \tau^4 E_4(\tau)\ ,\quad E_6(-1/\tau) = \tau^6 E_6(\tau)$

  causes modular anomaly of $Z_{(n_i)}(\tau, m, \epsilon_{1,2})$

- modular anomaly equation:

$$\frac{\partial}{\partial E_2} Z_{(n_i)}(\tau, m, \epsilon_{1,2} : E_2) = \frac{1}{24}\left[\epsilon_1\epsilon_2\Omega^{ij} n_i n_j - 2\Omega^{ij}(m^2 - \epsilon_+^2)\rho_i n_j\right] Z_{(n_i)}$$

$$\hat{Z}(\tau, v, m, \epsilon_{1,2}) = \sum_{n_1,\cdots,n_r=0}^{\infty} e^{-\sum_{i=1}^{r} n_i \alpha_i(v)} Z_{(n_i)}$$

$$\frac{\partial \hat{Z}}{\partial E_2} = \frac{1}{24}\left[\epsilon_1\epsilon_2\Omega^{ij}\partial_i\partial_j + 2(m^2 - \epsilon_+^2)\Omega^{ij}\rho_i\partial_j\right]\hat{Z} \equiv \frac{1}{24}\left[\epsilon_1\epsilon_2\Omega^{ij}\partial_i\partial_j + 2\Omega^{ij} I_i(m, \epsilon_+)\partial_j\right]\hat{Z}$$

- some manipulations: complete-square RHS

$$\frac{\partial Z_{\text{S-dual}}}{\partial E_2} = \frac{\epsilon_1\epsilon_2}{24}\Omega^{ij}\partial_i\partial_j Z_{\text{S-dual}}$$: heat equation

$$Z_{S-dual} \equiv \hat{Z}\ \boxed{\exp\left[\frac{I_i v^i}{\epsilon_1\epsilon_2} + \frac{\Omega^{ij} I_i I_j}{24\,\epsilon_1\epsilon_2} E_2(\tau)\right]}$$

$\rho^2 = \frac{c_2|G|}{12} \to \frac{N^3 - N}{12}$ for $G = A_{N-1}$

$$\exp\left[\frac{N^3 - N}{288}\frac{(m^2 - \epsilon_+^2)^2}{\epsilon_1\epsilon_2} E_2 - \varepsilon_0\right]$$

- modular anomaly: "standard" anomaly + anomaly of "standard" anomaly

[Nahmgoong, SK] (2017)

Z and f

- S-dual of $Z_{S\ dual}$ (satisfying 'heat equation') given by convoluting w/ Gaussian heat kernel ($\delta \equiv \frac{6}{\pi i \tau}$)

$$Z_{\text{S-dual}}\left(-\frac{1}{\tau}, v, \frac{m}{\tau}, \frac{\epsilon_{1,2}}{\tau}; E_2(-\frac{1}{\tau})\right) = Z_{\text{S-dual}}(\tau, v, m, \epsilon_{1,2}, E_2(\tau) + \delta)$$

$$Z_{\text{S-dual}}(\tau, v, m, \epsilon_{1,2}; E_2(\tau) + \delta) = \int_{-\infty}^{\infty} \prod_{i=1}^{N} dv_i' \; K(v, v') Z_{\text{S-dual}}(\tau, v', m, \epsilon_{1,2}; E_2(\tau))$$

$$K(v, v') = \left(\frac{i\tau}{\epsilon_1 \epsilon_2}\right)^{\frac{N}{2}} \exp\left[-\frac{\pi i \tau}{\epsilon_1 \epsilon_2}(v - v')^2\right]$$

- However, for many reasons, this expression is subtle:

- Firstly, this expression is "**wrong**".
- 4d limit: $K(v, v') = Z[S^3]$ of $T[SU(N)]$ QFT (S-duality domain wall)

  … motivated by AGT [Hosomichi, Lee, Park] [Drukker, Gaiotto, Gomis] [Teschner]
- Why wrong? Used 'wrong' (~Weyl asymmetric) perturbative part [Galakhov, Mironov, Morozov]
- Correction to Gaussian kernel is non-perturbative in $\epsilon_{1,2} \ll 1$: **prepotential OK**

  [→ Left-over question: 6d uplift of exact S-duality kernel & defect interpretation?]

- Also, often interested in regimes where other parameters get comparable to / larger than v, where we encounter **phase transitions**. **Easier to handle in prepotential**

Prepotential revisited

- 'S-dual' and 'anomalous' parts: Recall that

$$Z(\tau,v,m,\epsilon_{1,2}) = e^{-\varepsilon_0} Z_{U(1)}(\tau,m,\epsilon_{1,2})^N \sum_{n_1,\cdots,n_{N-1}=0}^{\infty} e^{-\sum_{i=1}^{N-1} n_i\alpha_i(v)} Z_{(n_i)}(\tau,m,\epsilon_{1,2}) \equiv e^{-\varepsilon_0} Z_{U(1)}^N \hat{Z}$$

$$\hat{Z} = Z_{\text{S-dual}} \exp\left[\varepsilon_0 - \frac{N^3-N}{288\epsilon_1\epsilon_2}(m^2-\epsilon_+^2)^2 E_2(\tau)\right]$$

- Saddle point approximation of S-duality relation for $Z_{S-dual} \sim \exp[-\frac{f\ (\tau,v,m)}{\epsilon_1\epsilon_2}]$:

$$\tau^2 f\left(-\frac{1}{\tau},\ v+\frac{1}{2\pi i\tau}\frac{\partial f}{\partial v},\ \frac{m}{\tau}\right) = f(\tau,v,m) + \frac{1}{4\pi i\tau}\left(\frac{\partial f}{\partial v}(\tau,v,m)\right)^2$$

- 'anomalous parts' in S-duality: remaining factors ~ $\exp[-\frac{F_{anom}(\tau,m)}{\epsilon_1\epsilon_2}]$

$$F_{\text{anom}} = N f_{U(1)}(\tau,m) + \frac{N^3-N}{288} m^4 E_2(\tau)$$

\- Both terms can be separately S-dualized easily, using $E_2(-1/\tau) = \tau^2\left(E_2 + \frac{6}{\pi i\tau}\right)$ and

$$f_{U(1)} = m^2\left(\frac{1}{2}\log m - \frac{3}{4} + \frac{\pi i}{2} + \log\phi(\tau)\right) + \sum_{n=1}^{\infty} \frac{m^{2n+2}B_{2n}}{2n\cdot(2n+2)!} E_{2n}(\tau)$$

Asymptotic free energy of 6d (2,0) theories

- Strategy: use "dual weak-coupling setting". anomalous part + 5d perturbative part

$$\tau^2 f\left(\boxed{-\frac{1}{\tau}}, \boxed{v+\frac{1}{2\pi i\tau}\frac{\partial f}{\partial v}}, \boxed{\frac{m}{\tau}}\right) = f(\tau,v,m) + \frac{1}{4\pi i\tau}\left(\frac{\partial f}{\partial v}(\tau,v,m)\right)^2$$

$\tau_D \to i0^+$ $\quad v_D$=fixed $\quad m_D$=fixed

- Want to keep 6d scalar small, $\Phi_D \ll T^2$: works if we keep v_D = finite
- $m_D \to 0$ is maximal SUSY enhancement point, $f = 0$: keep it finite to obstruct cancelation
- S-duality makes calculus easy if f on RHS can be approximated by f_{pert} (not always true)

- Results in "easy regions":

$$-\log Z \sim \frac{(f+f_{\rm anom})(\tau_D \to 0, v_D, m_D)}{\epsilon_1\epsilon_2} \sim \frac{i}{\epsilon_1\epsilon_2\tau_D}\left[\boxed{\frac{N^3 m_D^4}{48\pi} - \frac{\pi N m_D^2}{12}} \mp \boxed{\frac{N^2 m_D^3}{12}}\right] \qquad \text{for} \quad 0 < \pm{\rm Im}(m_D) < \frac{2\pi}{N}$$

from F_{anom} $\qquad$ from "low T" dual perturbative part

$$-\log Z \sim \frac{(f+f_{\rm anom})(\tau_D \to 0, v_D, m_D)}{\epsilon_1\epsilon_2} \sim \frac{i}{\epsilon_1\epsilon_2\tau_D}\left[\frac{N^3 m_D^4}{48\pi} - \frac{\pi N m_D^2}{12}\right] \qquad \text{for} \quad {\rm Im}(m_D) = 0$$

- real m_D: Take high T & then large N. $-\log Z \propto N^3 m_D^4$. (Physics not very clear to me…)

Index → “partition function”…?

- Non-perturbative phase transitions on RHS at $\mathrm{Im}(m_D) \to \pm\frac{2\pi}{N}$
- Bose/Fermi cancelation maximally obstructed at $m_D = \pi i(odd\ integer)$

$$4\sinh\frac{x+\pi i}{2}\sinh\frac{x-\pi i}{2} = 4\cosh^2\frac{x}{2} = 2+e^x+e^{-x}$$

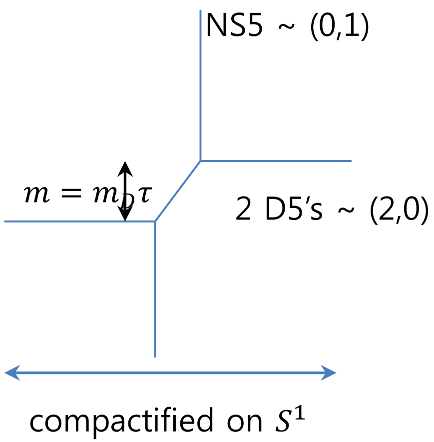
all states in the index counted w/ ‘+’ signs:
would maximize the “index entropy”

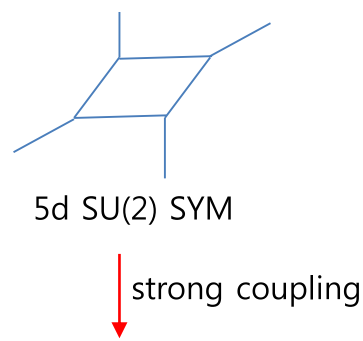
$$Z(\tau,m,\epsilon_{1,2},v) = Z_{\rm pert}(m,\epsilon_{1,2},v)\sum_{k=0}^{\infty}q^k Z_k(m,\epsilon_{1,2},v)$$

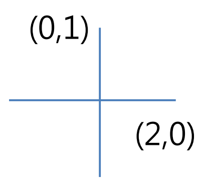
$$Z_k = \sum_{Y_i;\sum_{i=1}^N |Y_i|=k}\prod_{i,j=1}^{N}\prod_{s\in Y_i}\frac{\sinh\frac{E_{ij}(s)+m-\epsilon_+}{2}\sinh\frac{E_{ij}(s)-m-\epsilon_+}{2}}{\sinh\frac{E_{ij}(s)}{2}\sinh\frac{E_{ij}(s)-2\epsilon_+}{2}}$$

- A sequence of phase transitions: $m_D = 0, -\frac{2\pi i}{N}, -\frac{4\pi i}{N}, \ldots, -2\pi i$

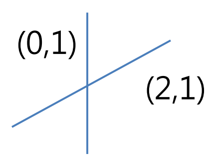
\- 2 M5’s on S^1 ~ 5d SU(2) $N = 1^*$ (w/ instanton corrections) ~ 2 D5’s forming web w/ NS5:



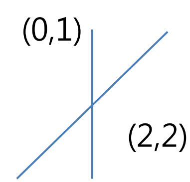




$m_D = 0$: perturbative singularity



$m_D = -\pi i$: 5d SCFT

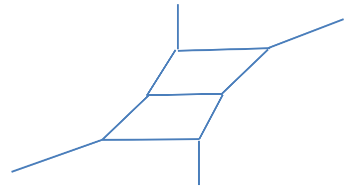


$m_D = -2\pi i \sim 0$

Phase transitions ~ 5d SCFTs

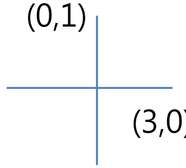
- More phase transition sequences given by 5d SCFTs:

\- 3 M5's

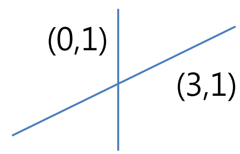


5d SU(3) SYM at level 0

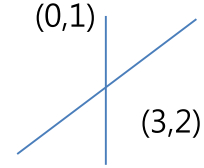
strong coupling



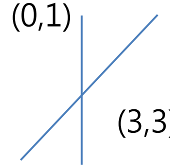
m=0: perturbative singularity



$m = -\frac{2\pi i}{3}$: 5d SCFT



$m = -\frac{4\pi i}{3}$: 5d SCFT



$m = -2\pi i \sim 0$

\- Similar (but more nontrivial) sequences of 5d SCFTs for $N \geq 4$ M5's

It apparently appears challenging to penetrate through the non-perturbative phase transitions to arrive at $m = -\pi i$, esp. at large N, but we are getting some progress… (and puzzles…)

A check

- Despite all the possible phase transitions, one can show that the anomalous part $\propto N^3 m_D^4$ is robust and is a direct consequence of 6d 't Hooft anomaly

- A check: [Di Pietro, Komargodski] [Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma]
  Some physics for "high T ~ small temporal S^1" admits 5d background fields' EFT

- 5d reduction of 6d Omega background (m, τ below mean m_D, τ_D… sorry…!)

$$ds^2(\mathbb{R}^4 \times T^2) = \sum_{a=1,2} \left| dz_a - \frac{2i\epsilon_a}{\beta} z_a dy \right|^2 + (dx - \mu dy)^2 + dy^2 = e^{2\phi}(dy + a)^2 + h_{ij} dx^i dx^j$$

$$a = \frac{1}{1 + \mu^2 + \frac{4\epsilon_a^2 |z_a|^2}{\beta^2}} \left(-\mu dx - \frac{2\epsilon_a |z_a|^2}{\beta} d\phi_a \right)$$

$$\tau = \frac{\beta}{4\pi}(\mu + i) \to 0$$

- gauge field in $U(1) \subset SU(2)_L \subset SO(5)_R$: $A_6 = \frac{2m}{\beta}$ $\mathcal{A} = -A_6 a$

- 5d CS terms determined by 6d U(1) anomaly $(2\pi)^4 I_8 \to \frac{N^3}{24} F^4$

$$S_{\text{CS}}^{(2)} = -\frac{iN^3 r_1}{96\pi^2} \int \left(A_6^4 a \wedge da \wedge da + 4A_6^3 \mathcal{A} \wedge da \wedge da + 6A_6^2 \mathcal{A} \wedge d\mathcal{A} \wedge da + 4A_6 \mathcal{A} \wedge d\mathcal{A} \wedge \mathcal{A} \right)$$

- One can argue $O(m^4)$ of $Im(-\log Z)$ is given solely from this CS term.

- Actual calculus completely agrees w/ S-duality-based studies (robust against phase transitions)

$$\text{Im}(S_{\text{eff}})\Big|_{m^4} = -i \frac{N^3 m^4 \mu}{12\epsilon_1 \epsilon_2 \beta(1 + \mu^2)}$$

Generalization to 6d (1,0) SCFTs

- A large part of the analysis for A_{N-1} (2,0) theory is extended to all (1,0) SCFTs.

- S-duality anomaly ← modular anomaly of elliptic genus for wrapped strings
- Modular anomalies ← 2d 't Hooft anomalies on 6d self-dual strings
- 2d anomaly ← 6d anomaly: inflow [H.-C.Kim, SK, Park] [Shimizu, Tachikawa] (2016)
- 6d anomaly in tensor branch [Ohmori, Shimizu, Tachikawa, Yonekura] [Intriligator]

$$I_8 = I_8^{\text{1-loop}} + \frac{1}{2}\Omega^{ij} I_i \boxed{I_j}$$

Green-Schwarz 4-form for classical anomaly in tensor branch (quadratic in field strengths $\{F\}$)

- Result (prepotential): $-\log Z(\tau, v, \{m\}) = \dfrac{f(\tau, v, \{m\}) + F_{\text{anom}}(\tau, \{m\})}{\epsilon_1 \epsilon_2}$

$$\tau^2 f\left(-\frac{1}{\tau},\ v + \frac{1}{2\pi i \tau}\frac{\partial f}{\partial v},\ \frac{m}{\tau}\right) = f(\tau, v, m) + \frac{1}{4\pi i \tau}\left(\frac{\partial f}{\partial v}(\tau, v, m)\right)^2$$

$$F_{\text{anom}}(\tau, m) = r f_{U(1)}(\tau, m) + f_{\text{vec+hyper}} + \frac{\Omega^{ij} I_i(m) I_j(m)}{24} E_2(\tau)$$

GS 4-form w/ $\{F\}$ replaced by corresponding masses

r: number of tensor multiplets

Ω^{ij}: tensor multiplets' kinetic term matrix $(i, j = 1, \ldots, r)$

- Can we use it to study high T behaviors of (1,0) theories?

- If there are 5d "dual weakly-coupled regimes" at small S^1. But usually, one finds 5d SCFTs.
- But certain Wilson line compactifications yield weakly-coupled 5d SYM

Concluding remarks

- S-duality (& its anomaly) of 6d $N=(2,0)$ SCFTs on T^2 (in tensor branch)
- Can generalize to general 6d $N=(1,0)$ SCFTs
- S-duality anomaly is closely related to 't Hooft anomalies
- N^3 at $T \gg 1/R_s$: growth of 5d KK fields for light D0. Partly explored but needs more study
- Computed $Im[f(\tau \to 0)] \propto N^3 m^4$ part directly from 6d anomaly

- D0-branes are key ingredients of IIA string theory which construct M-theory.
- It is natural to find their crucial roles also with M5 & N^3 d.o.f.

- Future directions:

- accounting for enhanced d.o.f. (e.g. by studying $m = -\pi i$ point)
- Non-perturbative S-duality (with defect operators)
- $SL(2,Z)$ of (2,0) & (1,1) little strings on $R^4 \times T^2$ [Hollowood, Iqbal, Vafa] [J.Kim, SK, K.Lee]
- ……